Topic.. Date..............................

Course Title : Numerical and Statistical Computing

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Programme : MCA (V sem)

Maximum marks : 100

Ques 1 : Obtain the roots of the equation $x^2 - 1 = 0$ by Regular falsi method.

Ans1.

$$f(x) = x^2 - 1$$

By Regula - falsi method -

$$f(0) = -1$$
$$f(1) = 1 - 1 = 0$$
$$f(2) = 4 - 1 = 3$$

Or

$$f(x) = x^2 - 1$$
$$\text{or } (x+1)\ (x-1)$$

Roots b/w lies $(0,2)$ then

we have $x_0 = 0$ $x_1 = 2$

then

$$x_2 = \frac{x_0 f(2) - x_1 f(0)}{f(2) - f(0)}$$

$$= \frac{0 \times 3 - 2 \times -1}{3+1} = \frac{2}{4} = \frac{1}{2} = 0.5$$

$$x_3 = \frac{x_2 f(2) - x_1 f(0.5)}{f(2) - f(0.5)}$$

$$= \frac{0.5 \times 3 - 2 \times -0.75}{3 + 0.75}$$

$$= \frac{1.5 + 1.5}{3.75} = \frac{3}{3.75} = \frac{300}{375} = 0.8$$

$$x_4 = \frac{0.8 \times 3 - 2 \times -0.36}{3 + 0.36} = \frac{2.4 + 0.72}{3.36} = \frac{3.12}{3.36} = 0.9286$$

Teacher's Sign..........................

Topic...Date...............................

Ques2. Apply Gauss - Elimination method to solve the following sets of equation

$x+4y-z=5$; $x+y-6z=12$; $3x-y-z=-4$

Ans2. Gauss - Elimination method-

$$-x+4y-z=-5 \quad \text{--- } ①$$

$$x+y-6z=-12 \quad \text{--- } ②$$

$$3x-y-z=-4 \quad \text{--- } ③$$

Pivot equation $= 3x-y-z=-4$

Step I =

$$\begin{bmatrix} 1 & 4 & -1 \\ 1 & 1 & -6 \\ 3 & -1 & -1 \end{bmatrix} = \begin{bmatrix} -5 \\ -12 \\ -4 \end{bmatrix}$$

$R_1 \rightarrow R_1 - R_2$

$$\begin{bmatrix} 0 & 3 & 5 \\ 1 & 1 & -6 \\ 3 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 7 \\ -12 \\ -4 \end{bmatrix}$$

$C_2 = C_2 - C_1$

$$\begin{bmatrix} 0 & 3 & 5 \\ 1 & 0 & -6 \\ 3 & -4 & -1 \end{bmatrix} = \begin{bmatrix} 7 \\ -12 \\ -4 \end{bmatrix}$$

$R_2 = R_3 - 3R_2$

$$\begin{bmatrix} 0 & 3 & 5 \\ 0 & -4 & 17 \\ 3 & -4 & -1 \end{bmatrix} = \begin{bmatrix} 7 \\ -12 \\ -4 \end{bmatrix}$$

$R_3 = R_3 - R_2$

Teacher's Sign..........................

$$\begin{bmatrix} 0 & 3 & 5 \\ 0 & -4 & 17 \\ 3 & 0 & -18 \end{bmatrix} = \begin{bmatrix} 7 \\ -12 \\ -4 \end{bmatrix}$$

$R_3 = \frac{1}{3} R_3$

$$\begin{bmatrix} 0 & 3 & 5 \\ 0 & -4 & 17 \\ 1 & 0 & -6 \end{bmatrix} = \begin{bmatrix} 7 \\ -12 \\ -4/6 \end{bmatrix}$$

$$3y + 5z = 7$$
$$-4y + 17z = -12$$
$$+x - 6z = -4/6$$

Solving these equations -

$$x_1 = \frac{-29}{15}$$

$$x_2 = \frac{12}{15}$$

$$x_3 = 2/3$$

Ques3. Use method of Lagrange's interpolation to find $f(0.16)$ for given function $f(x) = \sin(x)$ where $f(0.1) = 0.09983$ $f(0.2) = 0.19867$
Also. Find error in $f(0.16)$.

Ans3. Use the Lagrange's interpolation formula:-

$$f(0.16) = ?$$

$f(x) = \sin x$ $\qquad f(0.1) = 0.09983$

$$f(0.2) = 0.19867$$

Topic...Date.............................

$$f(0.16) = \frac{0.16 - 0.2\,(0.09983)}{0.1 - 0.2} + \frac{0.16 - 0.1\,(0.19867)}{0.2 - 0.1}$$

$$= 0.039932 - 0.119202$$

$$= 0.159134$$

The error of $f(0.16) = (0.00125)(0.19867)$

$$= \underline{\underline{0.00025}}.$$

Ques4. Find Newtons Forward difference interpolating polynomial for the following data –

x	0.1	0.2	0.3	0.4	0.5
f(x)	1.40	1.56	1.76	2.0	2.28

Ans4. Newton – Forward difference formula –

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0.1	1.40				
		0.16			
0.2	1.56		0.4		
		0.20		0	
0.3	1.76		0.4		0
		0.24		0	
0.4	2.00		0.4		
		0.28			
0.5	2.28				

Teacher's Sign.........................

Topic...Date..............................

$$a + hu = 0.6$$
$$0.3 + 1 \times u = 0.6$$
$$u = 0.3$$
$$x = 0.3 + 0.4 + 0.5 = \frac{1.2}{2} = 0.6$$

$$f(0.6) = f(0.3) + u\,\Delta f(0.3) + \frac{u(u-1)}{2}\Delta^2 f(0.3)$$
$$= 1.76 + 0.3 \times 0.24 + \frac{0.3(0.3-1)}{2}\Delta^2 \times 0.4$$
$$= 1.76 + 0.072 + \frac{0.3 \times -0.7}{2} \times 0.4$$
$$= 1.832 - 0.042$$
$$= 0.790$$

Ques5 Calculate the value of integral: $\int_0^6 \frac{dx}{1+x^2}$ by

i) Simpson's 1/3 rule.
ii) Simpson's 3/8 rule.

Ans5'
$$f(x) = \int_0^6 \frac{dx}{1+x^2} \text{ by}$$

i) Simpson's $\frac{1}{3}$ rule -

$$f(x) = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_6) + 2(y_2 + y_4)\right]$$

$$\int_0^6 \frac{dx}{1+x^2} = \frac{1}{3}\left[1 + \frac{1}{37} + 4\left(\frac{1}{2} + \frac{1}{10} + \frac{1}{26}\right) + 2\left(\frac{1}{5} + \frac{1}{17}\right)\right]$$
$$= 1.36617$$

Teacher's Sign..........................

Topic..Date...............................

ii) Simpson's $\frac{3}{8}$ rule-

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2y_3\right]$$

$$= \frac{3}{8}\left[1+\frac{1}{37}+3\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{17}+\frac{1}{26}\right)+2\left(\frac{1}{10}\right)\right]$$

$$\Rightarrow 1.35708$$

Ques5. A farmer buys a quantity of cabbage seeds from a company that claims that approximately 90% of the seeds will germinate if planted properly. If four seeds are planted, what is the probability that exactly two will germinate

Ans6. Acc. to given condition - $n=4$

$$P = \frac{90}{100} \rightarrow \frac{9}{10}$$

$$P[X=2]$$

Bionomial Theorem =

$$= {}^4C_2 \times \left(\frac{9}{10}\right)^2 \times \left(\frac{1}{10}\right)^2$$

$$= \frac{4!}{2!2!} \times \frac{81}{100} \times \frac{1}{100}$$

$$= \frac{4\times \cancel{2!}\times 3}{\cancel{2!}\times 2\times 1} \times \frac{81}{100} \times \frac{1}{100}$$

$$= \frac{486}{10000} = 0.0486$$

Teacher's Sign..........................

Topic.. Date..............................

Ques 7. Suppose that the amount of time one side spends in a bank to withdraw cash from an evening counter is exponentially distributed with mean ten minutes that is $\lambda = 1/10$. What is the probability that the customer will spend more than 15 minutes in the counter.

Let the amount of time = X

$$P(X>15)$$

then

$$= \int_{15}^{\infty} e^{-15}\lambda = e^{3/2}$$

$$P(X>15) = 0.223 \quad \text{or} \quad 22.3\%$$

Ques 8. Fit a straight line to the following data by the method of least square-

X	0	1	2	3	4
Y	1	1.8	3.3	4.5	6.3

Ans 8.

x	y	xy	x^2
0	1	0	0
1	1.8	1.8	1
2	3.3	6.6	4
3	4.5	13.5	9
4	6.3	25.2	16
$\Sigma x = 10$	$\Sigma y = 16.0$	$\Sigma xy = 4.71$	$\Sigma x^2 = 30$

Teacher's Sign..........................

Topic...Date...............................

$$y = ax+b$$
$$16 = 10a+5b \quad \text{—— (1)}$$
$$47.1 = 30a + 10b \quad \text{— (2)}$$

solving these equations -

we have multiply by (2) of eq (1) -

$$32 = 20a + 10b \quad \text{— (3)}$$

Subtraction (3) from (2) →

$$30a + \cancel{10b} = 47.1$$
$$-20a \pm \cancel{10b} = -32.0$$

$$10a = 15.1$$
$$a = 1.51$$

Put the value of a in eq (1) then

$$16 = 15.1 + 5b$$
$$5b = 0.9$$
$$b = \frac{0.9}{5} = 0.18$$
$$b = 0.18$$

Now put the value of a and b in

$$y = ax+b$$
$$\boxed{y = 1.51x + 0.18}$$

This is a fit straight line.

Teacher's Sign...........................

Qus9. compute approximate derivatives of $f(x) = x^2$ & $x = 0.5$ for the increasing value of h from 0.01 to 0.03 with a step size of 0.005 using
i) First order forward difference model.
2) First order backward difference model.

Ans9. i) $f(x) = x^2$
at $x = 0.5$
$h = 0.005$

The first order forward difference model.

$$= \frac{f(x+h) - f(x-h)}{2h}$$

$$= \frac{f(0.5+0.005) - f(0.5-0.005)}{2 \times 0.005}$$

$$= \frac{f(0.505) - f(0.495)}{0.010}$$

$$= \frac{(0.505)^2 - (0.495)^2}{0.010}$$

$$= \frac{0.255 - 0.245}{0.010}$$

$$= \frac{0.010}{0.010} = 1$$

ii) First order backward difference model.

$$f'(x) - \frac{f(x-h)}{y' - (x-h)}$$

$$= \frac{2x - f(0.405)}{1 - 0.495}$$

$$= \frac{2 \times 0.5\ \ 1 - 0.245}{0.505} = \frac{1 - 0.245}{0.505}$$

Topic.. Date..........................

$$= \frac{0.755}{0.505} = 0.149.$$

Qus10. Find root of eqn $x^3 - x - 1 = 0$ lying b/w 1 & 2 by bisection method.

Ans10.
$f(x) = x^3 - x - 1 = 0$

$f(1) = 1-1-1 = -1 \Rightarrow$ -ive

$f(2) = 8-2-1 = 5$ (+ive)

The roots lies b/w 1 & 2

$f(1.324) = -0.00306 = $ -ive

$f(1.325) = 0.00120 = $ +ive

Hence, Roots lies b/w 1.324 − 1.325

$$x_2 = \frac{1.324 + 1.325}{2} = 1.32475$$

$f(x_2) = 0.000136$ i.e +ive

Now Roots b/w 1.3245 & 1.32475

$$x_3 = \frac{1.3245 + 1.32475}{2} = 1.324625$$

$f(x_3) = $ -ive

$$x_4 = \frac{1.324625 + 1.32475}{2} = 1.324875$$

$f(x_4) = $ -ive

the roots lies b/w 1.3246875 & 1.32475

$$x_5 = \frac{1.3246875 + 1.32475}{2}$$

$$= 1.32471875$$

Teacher's Sign..........................

Topic...Date................................

Qus11. A problem in statistics is given to three students A, B & C whose chance of solving it are $1/2$, $3/4$ & $1/4$ respectively. What is the probability that the problem will be solved?

Ans11. Let A, B and C be the respective event of solving the problem and $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$

Then A, B, C are independent event -

$$P(A) = \frac{1}{2}$$

$$P(\bar{B}) = \frac{3}{4}$$

$$P(\bar{C}) = \frac{1}{4}$$

$$P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A})\ P(\bar{B})\ P(\bar{C})$$

$$= \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} = \frac{3}{32}$$

Non - solve problem $= 1 - \frac{3}{32} = \frac{29}{32}$ Ans

Qus12. In a partially destroyed laboratory the record of an analysis of correlation data was found the following results are legible -

Variance of $x = 9$

Regression equations $= 8x - 10y + 66 = 0$

$40x - 18y - 214 = 0$

Find -

i) The mean values of x & y.

Topic...Date.............................

ii) The correlation b/w x & y.
iii) Standard deviation of y.

Ans 12. i) $8x - 10y + 66 = 0$ —— ①
$40x - 18y - 214 = 0$ —— ②

solve the eq ① & ② then we get -
$x = 13$
$y = 17$
be the lines of regression of y and x.

ii) x on y then
$$y = \frac{8}{10}x + \frac{66}{10}$$
$$x = \frac{18}{40}y + \frac{214}{40}$$
$$x \text{ on } y = \frac{\cancel{18}}{\cancel{40}} = \frac{9}{20}$$
$$y \text{ on } x = \frac{\cancel{8}}{\cancel{10}} = \frac{4}{5}$$
$$r^2 = \frac{9}{\cancel{20}5} \times \frac{\cancel{4}1}{5} = \frac{9}{25}$$
$$= \pm\frac{3}{5} = 0.6$$

iii) Standard Deviation ⇒
$$x \text{ on } y = \frac{r\sigma y}{\sigma x} = \frac{0.6 \times 10}{8} = \frac{6}{8} = \frac{3}{4}$$
$$y \text{ on } x = \frac{0.6 \times 40}{18} = \frac{24}{\cancel{18}} = \frac{8}{6} = \frac{4}{3}$$

Teacher's Sign...........................

Topic..Date..............................

Ques13. An individual IQ score has a Normal distribution N(100, 152). Find the probability that an individual IQ ~~soro~~ score is b/w 91 & 21.

Ans13 We know

$$P[91 < x < 121]$$

$$= P\left[\frac{91-100}{15} < \frac{x-100}{15} < \frac{121-100}{15}\right]$$

$$= P\left[\frac{-9}{15} < \frac{x-100}{15} < \frac{21}{15}\right]$$

$$= P[-0.6 < z < 1.4]$$

$$= 0.9192 - 0.2743 = 0.6449$$

Ques14. Solve the initial value problem $\frac{dy}{dx} = y - x$ with $y=(0) = 2$ and $h = 0.1$. Using fourth order classical Runge - Kutta method. Find $y(0.1)$ & $y(0.2)$ correct to four decimal places.

Ans14.

$$\frac{dy}{dx} = y - x$$

$$y(0) = 2$$

$$h = 0.1$$

given that $\frac{dy}{dx} = y - x$

taking $h = 0.1$ $\quad x_1 = x_0 + h$

$$x_0 = 0$$

$$x_1 = 0 + 0.1 = 0.1$$

Teacher's Sign..............................

Topic.. Date..............................

$$x_2 = x_1 + h = 0.1 + 0.1 = 0.2$$

By Runge-Kutta method

$$y_{n+1} = y_n + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right] \quad (1)$$

whose $k_1 = h\, f(x_n, y_n)$

$$k_2 = h\, f\left(x_n + \frac{h_1}{2}, y_n + \frac{h_1}{2}\right)$$

$$k_3 = h\, f\left(x_n + \frac{h_2}{2}, y_n \frac{k_2}{2}\right)$$

$$k_4 = h\, f\left(x_n + h + y_n + k\right)$$

Put $n=0$

$$k_1 = 0.1\, f(x_0, y_0)$$
$$= 0.1\, \{y_0 - x_0\}$$

$$= 0.1\,(2-0) = 0.2$$

$$k_2 = 0.1\,(y_1 - x_1)$$

$$y_1 = y_0 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$= 2 + \frac{1}{6}\left[0.2 + 2k_2 + 2k_3 + k_4\right]$$

$$k_2 = 0.1\left[0 + 0.5, 2 + 0.1\right\}$$

$$k_2 = 0.1\left[2.1 - 0.5\right)$$

$$= 0.1 \times 1.6 = 0.16$$

Teacher's Sign..............................

$k_3 = 0.1 \left[(2+0.2 - 0+0.2 \right]$

$= 0.1 \left[2.2 \mp 0.2 \right] = 0.2$

$\Rightarrow$ Put the value of k_1, k_2, k_3, k_4

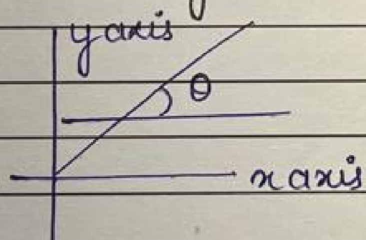
$y_1 = 2 + \frac{1}{6} \left[0.2 + 2 \times 0.1 + 2 \times 0.2 + 0.2 \right]$

$= 2 + \frac{1}{6} \left(0.2 + 0.2 + 0.4 + 0.2 \right]$

$= 2 + \frac{1}{6} = \frac{13}{6} = 2.16.$

Ques15. The tangent of angle b/w the lines of regression y on x and x on y is 0.6. & and $\sigma_x = \frac{1}{2}\sigma_y$. Find r_{xy}.

Ans15.



given that -

line of regression $r = 0$

then

x on y $\Rightarrow$

$(x - \bar{x}) = b_{xy} (y - \bar{y})$

$x - \bar{x} = \frac{r \sigma_x}{\sigma_y} (y - \bar{y})$

$y - \bar{y} = \frac{\sigma_y}{r \sigma_x} (x - \bar{x})$

Topic..Date...............................

$$\boxed{m_1 = \frac{\sigma_y}{r\sigma_x}}$$

when y on $x \Rightarrow$

$$(y-\bar{y}) = b_{yx}(x-\bar{x})$$

$$y-\bar{y} = r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$$

$$\boxed{m_2 = r\frac{\sigma_y}{\sigma_x}}$$

we know that -

$$\tan\theta = \pm\frac{m_1 - m_2}{1+m_1 m_2}$$

$$\tan\theta = \pm\frac{\frac{\sigma_y}{r\sigma_x} - r\frac{\sigma_y}{\sigma_x}}{1+\frac{\sigma_y}{\cancel{r}\sigma_x}\times\cancel{r}\frac{\sigma_y}{\sigma_x}}$$

$$= \pm\frac{\frac{\sigma_y - r^2\sigma_y}{r\sigma_x}}{1+\frac{\sigma_y\times\sigma_y}{(\sigma_x)^2}} = \pm\frac{\sigma_y - r^2\sigma_y}{r\sigma_x}\times\frac{(\sigma_x)^2}{(\sigma_x)^2+(\sigma_y)^2}$$

$\Rightarrow$ when we put $r=0$

then we find $\rightarrow$ $\underline{\tan\theta = \infty}$

$$\boxed{\theta = \pi/2}$$

Teacher's Sign...........................

Topic.. Date..............................

Qus16. Evaluate the integral $I = \int_0^{\pi/2} \sin x \, dx$ using gauss-Lengendre formula. compare the results with exact solution by Simpson rule. The exact value of $I = 1$

Ans16. $I_1 = \int_0^{\pi/2} \log \sin x \, dx$

Formula –

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$I_2 = \int_0^{\pi/2} \log \left[\sin\left(\frac{\pi}{2} - x\right) dx\right]$$

$$= \int_0^{\pi/2} \log \cos x \, dx$$

$$I_1 + I_2 = \int_0^{\pi/2} \log \sin x \, dx + \int_0^{\pi/2} \log \cos x \, dx + \int_0^{\pi/2} \log 2 \, dx - \int_0^{\pi/2} \log 2 \, dx$$

$$2I = \int_0^{\pi/2} (\log \sin x + \log \cos x + \log 2)\, dx - \int_0^{\pi/2} \log 2 \, dx$$

$$= \log a + \log b = \log (ab)$$

$$2I = \int_0^{\pi/2} \log 2 \,(\sin x \cos x)\, dx$$

$$2I = \int_0^{\pi/2} \log \sin 2x \, dx - \int_0^{\pi/2} \log 2 \, dx$$

$$= \int_0^{\pi/2} \log \sin 2x \, dx - \log 2 \int_0^{\pi/2} 1 \cdot dx$$

Teacher's Sign..........................

Topic..Date..................................

$$= \int_0^{\pi/2} \log \sin 2x \, dx - \log 2 \left(\frac{\pi}{2} - 0\right)$$

Put $2x = t$

$2\,dx = dt$

$2 = \frac{dt}{dx}$ where $= x = 0, \; t = 0$

$x = \frac{\pi}{2} \quad t = \pi$

$$2I = \int_0^{\pi} \log \sin t \, dt - \log 2 \left(\frac{\pi}{2}\right)$$

$$= \int_0^{2a} f(x)\,dx = \int_0^{a} f(x) + \int_0^{a} f(2a-x)\,dx$$

$$2I = \frac{\pi}{2}\left[\int_0^{\pi/2} \log \sin t \, dt + \int_0^{\pi/2} \log \sin t \, dt\right] - \frac{\pi}{2} \log 2$$

$$= \frac{1}{2}\left(\int_0^{\pi/2} \log \sin t \, dt + \int_0^{\pi/2} \log \sin t \, dt - \frac{\pi}{2} \log 2\right.$$

$$= \frac{1}{2}\left\{2 \int_0^{\pi/2} \log \sin t \, dt\right\} - \frac{\pi}{2} \log 2$$

$$2I = \int_0^{\pi/2} \log \sin t \, dt - \frac{\pi}{2} \log 2$$

$$I = \int_0^{\pi/2} \log \sin t \, dt - \int_0^{\pi/2} \log 2 \, dt$$

Teacher's Sign..........................